TIME-DELAY EFFECT AND ITS SOLUTION FOR OPTIMAL OUTPUT FEEDBACK CONTROL OF STRUCTURES

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SUMMARY

This paper investigates the stability of MDOF optimal direct output feedback control systems through analysis of system modal properties after the application of time-delayed control force. Explicit formula and numerical solution are obtained to determine the maximum delay time and critical delay time which cause system instability and control ineffectiveness, respectively. The results indicate that direct velocity feedback has longer maximum and critical delay times than state feedback. The feedback of non-collocated measurements will reduce maximum delay time. The ratios of maximum and critical delay times to structural natural period decrease as the active damping increases. For a given damped structure, a critical control weighting factor exists. When a larger control weighting factor is used, the control system will remain stable even with longer delay time. A formula is also developed to determine the critical control weighting factor so as to make the stability of MDOF control systems dominated by lower modes. Hence, the maximum delay time and critical delay time can be significantly lengthened by selecting an appropriate control weighting factor and/or adding higher modal dampings.

KEY WORDS: active control; direct output feedback; time-delay solution; earthquake engineering

1. INTRODUCTION

In recent years, considerable research effort has been devoted to the development and application of active control to reduce dynamic responses and increase serviceability of civil engineering structures under environmental loadings such as wind and earthquake. It has reached the stage where active control systems have been installed in real structures and reported to be able to decrease the wind- and earthquake-induced responses significantly.[1–3] Active control is also accepted as a possible means for retrofitting or strengthening an existing structure. However, before active control systems can be widely used, there are still some practical and important problems to be solved, such as limited number of sensors and controllers and control force execution time delay. The detailed review, state-of-the-art of the progresses, and future direction in this field of research may be found in Soong,[4] Housner and Masri,[5,6] and Housner *et al.*[7]

Most previous research work employed state feedback control theory with the assumption of full-state measurements. However, a real civil engineering structure usually possesses a large number of degrees of freedom. Economy, data processing and on-line calculation make it impossible and impractical to acquire full-state measurements and feedback. Only output measurement, which is usually a combination of responses at a few degrees-of-freedom, is available for control force calculation. Thus, direct output feedback becomes necessary from a practical point of view. According to Chung *et al.*,[8] direct velocity feedback control is effective in reducing structural dynamic responses with a very small number of sensors and controllers compared with the number of degrees-of-freedom of the structure.

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In real active control systems, time is consumed in data acquisition, data processing, on-line calculation, and control force execution. There is always a delay between the time at which the control force is assumed to be applied and actually applied. Even though the delay time is much smaller than structural natural period and may be minimized by employing more advanced hardware and software, time-delay effect cannot be avoided and eliminated. Small delay time not only can render the control ineffective, but also may cause the system instability.[4] Thus, time-delay effect must be considered in control design before active control devices are implemented on real structures.

The time-delay effect on active control systems has been investigated by many researchers[9–15] and various methodologies to deal with this problem have also been available in the literature.[9, 10, 13, 16–20] Most previous research studied the stability for single or equivalent single degree-of-freedom (SDOF) systems based on state feedback. Most recently, Agrawal *et al.*[10] considered an undamped SDOF system and obtained its analytical expression of maximum allowable delay time beyond which the control system becomes unstable. Inaudi and Kelly[13] examined the time-delay effect on actively isolated structures based on velocity feedback and obtained an expression of maximum delay time as function of given active dampings. The time-delay effect on multiple degree-of-freedom (MDOF) damped structures based on optimal direct output feedback control algorithm and the selection of optimum control parameters to prevent control systems from instability due to time delay have not yet been investigated quantitatively.

In this paper, the stability of MDOF optimal output feedback control systems is investigated through analysis of system modal properties after the application of time-delayed control force. Explicit formula and numerical solution are obtained to determine the maximum delay time $t_{d,max}$ and critical delay time $t_{d,cr}$ which cause system instability and control ineffectiveness, respectively, for state feedback and direct velocity feedback control systems. Both $t_{d,max}$ and $t_{d,cr}$ depend on structural original frequencies, damping ratios and the selected control weighting factor. They are two important parameters for the design of actuators and the solutions to time-delay problem. It is found that direct velocity feedback has longer $t_{d,max}$ and $t_{d,cr}$ than state feedback. The feedback of non-collocated measurements will reduce $t_{d,max}$. $t_{d,max}$ and $t_{d,cr}$ increase as structural original damping increases but, decrease with the increase of structural original frequency as well as active damping due to the application of control forces. However, the ratios of maximum and critical delay times to structural natural period are nearly independent of structural original frequency and damping ratio for both state feedback and direct velocity feedback and, decrease as the control weighting factor decreases. For a given damped structure, a critical control weighting factor exists. When a larger control weighting factor is used, the control system will remain stable even with longer delay time. The critical control weighting factor decreases as structural original damping ratio increases. In this study, a formula is also developed to determine the critical control weighting factor so as to make the stability of MDOF control systems dominated by lower modes. Under this circumstance, the maximum and critical delay times can be significantly lengthened by selecting an appropriate control weighting factor and/or adding higher modal dampings.

2. TIME-DELAY EFFECT IN DIRECT OUTPUT FEEDBACK CONTROL

For an n-DOF structure under dynamic loading $\mathbf{w}(t)$ and active control force $\mathbf{U}(t)$, the state equation and output measurement equation are expressed, respectively, as

$$\dot{\mathbf{z}}(t) = A\mathbf{z}(t) + B\mathbf{U}(t) + E\mathbf{w}(t) \tag{1}$$

$$\mathbf{y}(t) = D\mathbf{z}(t) \tag{2}$$

where $\mathbf{z}(t)$ is a $2n \times 1$ state vector, A a $2n \times 2n$ system matrix, B a $2n \times q$ controller location matrix of q controllers, E a $2n \times r$ external loading location matrix of r loadings, $\mathbf{y}(t)$ a $p \times 1$ output vector from p sensors, $(p \ll 2n)$, and D a $p \times 2n$ output matrix.

In direct output feedback, the control forces are calculated directly from the multiplication of delayed output measurements by constant feedback gains

$$\mathbf{U}(t) = G\mathbf{y}(t - t_d) \tag{3}$$

where t_d is the delay time. G is a $q \times p$ time-invariant feedback gain matrix obtained in the case of no time delay. Based on the optimal direct output feedback control algorithm formulated in Reference 8, for any output measurement, an optimal output feedback gain matrix G exists and is found by minimizing the following quadratic performance index

$$J = \int_0^\infty [\mathbf{z}^T(t)Q\mathbf{z}(t) + \mathbf{U}^T(t)R\mathbf{U}(t)]\,dt \tag{4}$$

where Q and R are weighting matrices for the states and control forces, respectively. Their relative values indicate the degree of control forces to be applied. The system stability is examined through analysis of system modal properties after time-delayed control. Substituting equations (2) and (3) into equation (1), the state equation without external loading becomes

$$\dot{\mathbf{z}}(t) = A\mathbf{z}(t) + BGD\mathbf{z}(t - t_d) \tag{5}$$

The system poles or eigenvalues are then obtained by solving the following sets of homogeneous algebraic equations

$$|\lambda I - (A + e^{-t_d\lambda}BGD)| = 0 \tag{6}$$

where I is the identity matrix and λ represents complex eigenvalues of the time-delayed control system. The corresponding system frequency and damping ratio are given as

$$\omega_c = |\lambda|; \qquad \xi_c = -\mathrm{Re}(\lambda)/\omega_c \tag{7}$$

2.1. *MDOF active tendon structure*

For a MDOF structure with a set of four active tendons placed at the first floor, the control force $U(t)$ is expressed by

$$U(t) = 4k_c(\cos\alpha)u(t) \tag{8}$$

where $u(t)$ is actuator displacement, k_c and α are stiffness and inclination angle of the tendons, respectively. The weighting matrices are given as

$$Q = \begin{pmatrix} K & 0 \\ 0 & 0 \end{pmatrix}, \qquad R = \frac{\beta}{16k_c\cos^2\alpha} \tag{9}$$

to make performance index J represent the summation of structural strain energy and applied control energy. K is the structural stiffness matrix. The coefficient β, called control weighting factor, determines the relative importance of control effectiveness (response reduction) and economy (control force requirements). When $\beta < 1$, control effectiveness has more weight and, when $\beta > 1$, economy is more important. $\beta = \infty$ represents uncontrolled case.

The control effectiveness of direct output feedback systems may be investigated by varying the value of β. It has been verified that for small time delay, direct velocity feedback with one collocated controller and sensor is quite effective in reducing structural responses. However, when delay time increases, all modal active damping ratios, ξ_c, decrease. The smaller β is selected, the faster damping ratio reduces. The active damping of higher mode decreases faster than those of lower modes. For example, Figure 1 shows a 3-DOF active tendon structure with system properties listed in Table I. It is seen from Figure 2 that the third active damping ratio first drops to zero leading to the instability of whole control system. Hence, to assure control system

stability and control effectiveness, the knowledge of maximum allowable delay time and the ways to lengthen delay time are very important for active control design.

Table I. Parameter values of 3-DOF active tendon system

System parameter	Parameter values
Mass matrix, M (kg)	$\begin{bmatrix} 981 & 0 & 0 \\ 0 & 981 & 0 \\ 0 & 0 & 981 \end{bmatrix}$
Stiffness matrix, K (N/cm)	$\begin{bmatrix} 27417 & -16416 & 3691 \\ -16416 & 30222 & -16248 \\ 3691 & -16248 & 13336 \end{bmatrix}$
Damping matrix, C (N s/cm)	$\begin{bmatrix} 3.828 & -0.573 & 0.617 \\ -0.573 & 4.569 & -0.026 \\ 0.617 & -0.026 & 4.375 \end{bmatrix}$
Modal frequency, ω_0 (Hz)	$\begin{bmatrix} 2.24 \\ 6.80 \\ 11.49 \end{bmatrix}$
Modal damping ratio, ξ_0 (%)	$\begin{bmatrix} 1.61 \\ 0.39 \\ 0.36 \end{bmatrix}$
Tendon stiffness, k_c (N/cm)	3721
Tendon inclination, α (°)	36

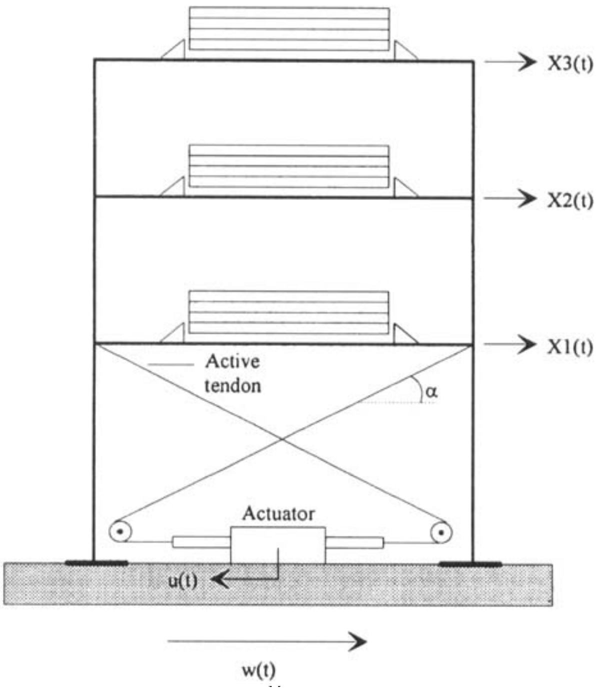
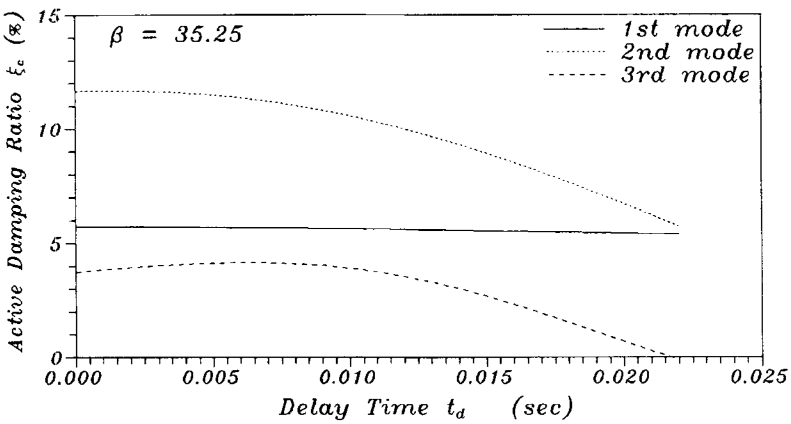


Figure 1. 3-DOF active tendon system



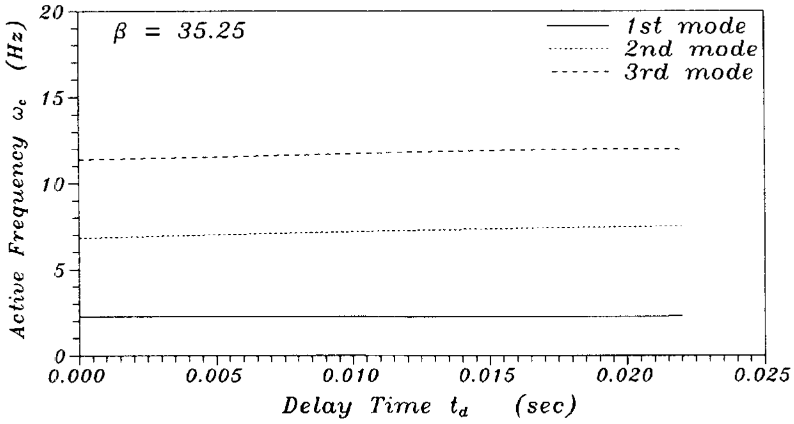


Figure 2. Time-delay effect on modal parameters of 3-DOF control system with one collocated velocity feedback

3. MAXIMUM DELAY TIME AND CRITICAL DELAY TIME

In this paper, $t_{\mathrm{d,max}}$ and $t_{\mathrm{d,cr}}$ are defined as maximum delay time and critical delay time which cause system instability ($\xi_c = 0$) and control ineffectiveness (ξ_c = original damping ratio, ξ_0), respectively. Assigning the real part of λ equal to zero and solving equation (6), the analytical expressions of active frequency and maximum delay time when control system becomes unstable are given as

$$\omega_{\mathrm{c,max}} = \left[\sqrt{c_3 + \sqrt{c_3^2 - 2c_1 + c_1^2}}\,\right]\omega_0 \tag{10}$$

$$t_{\mathrm{d,max}} = \frac{1}{\omega_{\mathrm{c,max}}}\cos^{-1}\left[\frac{1}{2} + \frac{1}{2}\frac{(1-c_1^2)}{(1-c_1)^2 + (2\xi_0 - 2\sqrt{c_1}\xi_{c0})^2\omega_{\mathrm{c,max}}^2/\omega_0^2}\right] \tag{11}$$

for a SDOF active tendon structure using state feedback. In above equations,

$$c_1 = \sqrt{1 + \frac{4}{\gamma}}\,; \quad \gamma = \left(\frac{k}{4k_c\cos^2\alpha}\right)\beta; \qquad c_3 = (1 - 4\sqrt{c_1}\xi_0\xi_{c0} + 2c_1\xi_{c0}^2) \tag{12}$$

ω_0 and k are structural original frequency and stiffness, respectively. ξ_{c0} is the active damping ratio as $t_d = 0$ and takes the form

$$\xi_{c0} = \frac{1}{2}\sqrt{\frac{4\xi_0^2 + 2(c_1 - 1)}{c_1}} \tag{13}$$

Similarly, for direct velocity feedback,

$$\omega_{c,\max} = \left[\sqrt{\xi_{c0}^2 + 1 - 2\xi_0\xi_{c0}} + \sqrt{\xi_{c0}^2 - 2\xi_0\xi_{c0}}\,\right]\omega_0 \tag{14}$$

$$t_{d,\max} = \frac{1}{\omega_{c,\max}}\cos^{-1}\frac{\xi_0}{\xi_0 - \xi_{c0}} \tag{15}$$

in which ξ_{c0} is expressed as

$$\xi_{c0} = \sqrt{\xi_0^2 - \frac{4\xi_0^2 - c_2}{c_2\gamma + 4}}; \qquad c_2 = 1 + \frac{1}{\omega_0^2} \tag{16}$$

Equations (11) and (15) are exact formulas of maximum delay time for a damped structure with active control. The critical delay times for state feedback and direct velocity feedback are obtained numerically. In the special case of an undamped structure ($\xi_0 = 0$) with velocity feedback, equation (15) reduces to

$$t_{d,\max} = t_{d,cr} = \frac{\pi}{2(\sqrt{\xi_{c0}^2 + 1} + \xi_{c0})\omega_0} \tag{17}$$

which is identical to that by Agrawal *et al.*[10] Since a real structure has always some damping, equations (11) and (15) are thus the general solutions of maximum delay times.

It is observed from equations (11), (15) and (17) that both $t_{d,\max}$ and $t_{d,cr}$ increase as structural original damping increases but decrease with the increase of structural original frequency as well as active damping by both state and velocity feedback controls, as seen in Figures 3 and 4. However, the ratios of maximum delay time and critical delay time to structural natural period T_0 are nearly independent of ω_0 and ξ_0. The ratios increase as β increases. For a given damped structure, a critical (or minimum) value of β exists. When a larger β is used, the control system will remain stable even with longer delay time as seen in Figure 5. In addition, it is found from Figure 5 that direct velocity feedback has longer $t_{d,\max}$ and $t_{d,cr}$ than state feedback. When β is large, the ratio of $t_{d,cr}/T_0$ approaches 0·25 for both state feedback

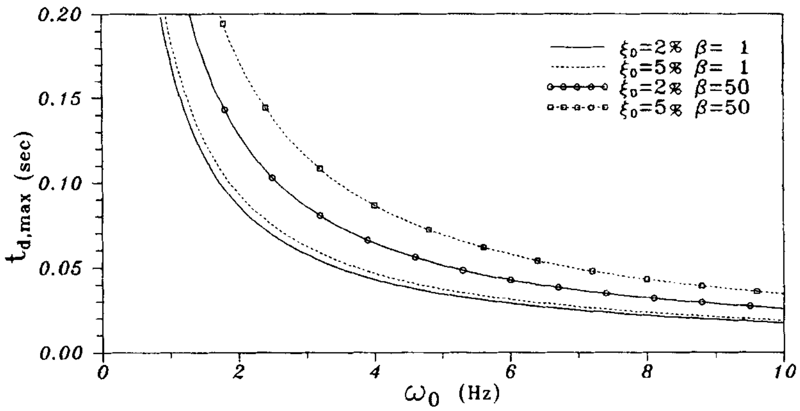


Figure 3. Maximum delay time of SDOF control system with velocity feedback

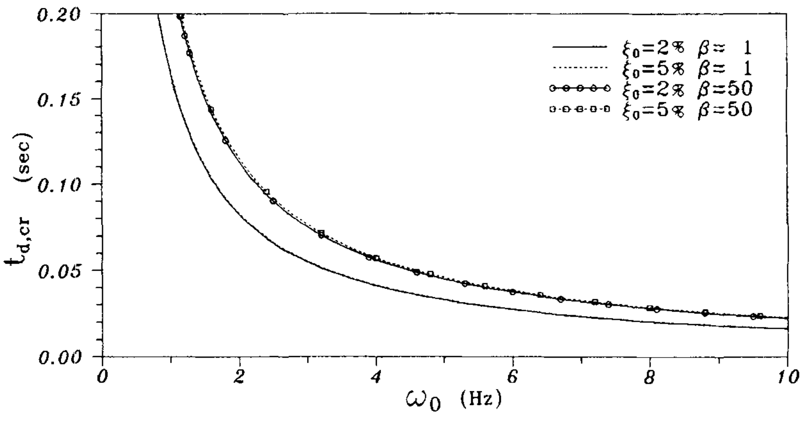


Figure 4. Critical delay time of SDOF control system with velocity feedback

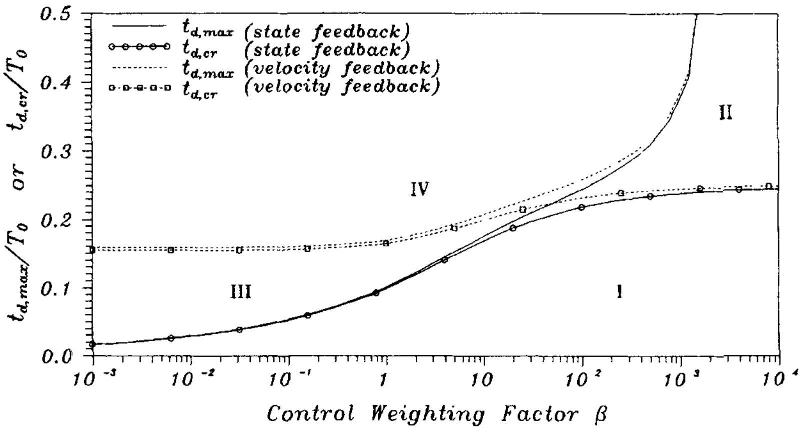


Figure 5. Delay-time ratios of state and velocity feedback control systems

and velocity feedback. When β is very small, $t_{d,max}/T_0$ and $t_{d,cr}/T_0$ approach 0·155 for direct velocity feedback. For a selection of β to make $\xi_{c0} = 10$ percent, if the ratio of $t_{d,max}/T_0$ or $t_{d,cr}/T_0$ falls in area I, the active control system is stable and effective for both state feedback and velocity feedback; in area II, the control system is stable but ineffective; in area III, the control system is stable and effective for velocity feedback but unstable for state feedback; and in area IV, both state and velocity feedback control systems are unstable.

To examine the influence of numbers and locations of sensors on the stability of the 3-DOF active control system, direct output feedback with various numbers and locations of sensors is investigated. The pattern of controller and sensor types and locations are symbolized by F, X, and V. For example, F1X1V1 indicates that one controller is placed on the first floor and the displacement and velocity for the first floor are measured. Table II shows velocity feedback with one collocated controller and sensor (F1V1) has longer $t_{d,max}$ than F1X1V1 and state feedback (F1State) as concluded in SDOF control systems. It indicates that the feedback of non-collocated measurements will reduce $t_{d,max}$. In general, $t_{d,max}$ is determined by the third (the highest) mode. However, when β or the third original frequency ($\omega_{0,3}$) and damping ratio ($\xi_{0,3}$) increase, the active damping ratio of the second or first mode will decrease to zero earlier than that of the third mode, as illustrated in Figure 6. Under this circumstance, $t_{d,max}$ will thus be increased. This finding eliminates the question that a real structure allows very small delay time because of inherent large frequencies in higher modes, and thus increases the confidence of the application of active control.

Table II. Control results of 3-DOF control system with various numbers of measurements as $\beta = 35.25$

Control cases	ω_{c0} (Hz)	ξ_{c0} (%)	$t_{d,max}$ (ms)
F1 State	$\begin{bmatrix} 2.26 \\ 6.87 \\ 11.51 \end{bmatrix}$	$\begin{bmatrix} 10.00 \\ 9.44 \\ 4.51 \end{bmatrix}$	19.0
F1X1V1	$\begin{bmatrix} 2.29 \\ 6.96 \\ 11.44 \end{bmatrix}$	$\begin{bmatrix} 5.74 \\ 11.30 \\ 3.88 \end{bmatrix}$	20.4
F1V1	$\begin{bmatrix} 2.25 \\ 6.83 \\ 11.40 \end{bmatrix}$	$\begin{bmatrix} 5.74 \\ 11.66 \\ 3.73 \end{bmatrix}$	21.7

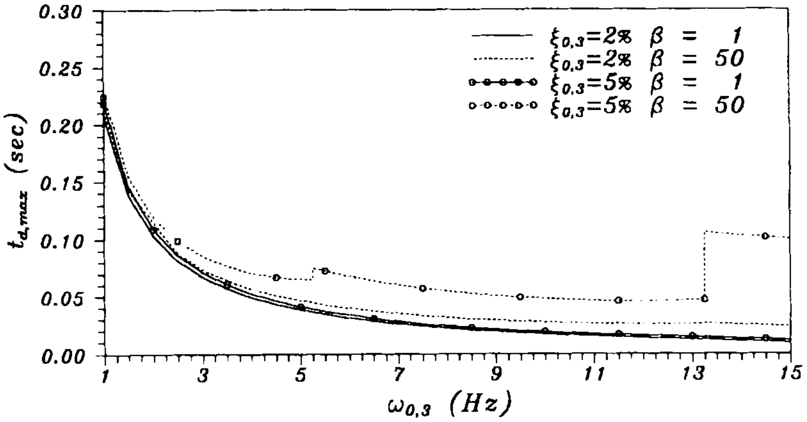


Figure 6. Effect of β and the third modal parameters on $t_{d,max}$ for 3-DOF control system with F1V1 feedback

4. CRITICAL CONTROL WEIGHTING FACTOR

As found in preceding section, the maximum delay time may be lengthened by selecting an appropriate control weighting factor and/or increasing the damping ratio of higher modes. For a given SDOF damped structure, a minimum (or critical) control weighting factor (β_{min}) exists to satisfy equations (11) or (15). That means the value of the function in arc cosine equals to -1. For direct velocity feedback,

$$\beta_{min} = \frac{1}{3\xi_0^2[k/(4k_c \cos^2\alpha)]} - \frac{16}{3c_2[k/(4k_c \cos^2\alpha)]} \tag{18}$$

which corresponds to $\xi_{c0} = 2\xi_0$. It indicates the control system will always be stable if $\xi_{c0} < 2\xi_0$ or say $\beta > \beta_{min}$. From equation (18), it is observed that β_{min} is independent of ω_0 as $\omega_0 \geqslant 0.5$ Hz. β_{min} decreases with the increase of ξ_0 as shown in Figure 7. With a selection of $\beta > \beta_{min}$ (zone I), the control system will be stable even with longer time delay. Direct velocity feedback has smaller β_{min} than state feedback. Therefore, its control performance could be better than that of state feedback.

For a 3-DOF active tendon structure, β_{min} versus $\xi_{0,3}$ for $\omega_{0,3} = 1, 5$, and 10 Hz is shown in Figure 8 with F1V1 feedback control. β_{min} decreases with the increase of $\omega_{0,3}$ and $\xi_{0,3}$. The larger $\omega_{0,3}$ or $\xi_{0,3}$ is, the smaller the β that can be used. For a given $\omega_{0,3}$, a critical $\xi_{0,3}$ exists in which β_{min} approaches zero. This critical $\xi_{0,3}$ decreases as $\omega_{0,3}$ increases. This reconfirms the fact that the stability of MDOF control systems turns to be dominated by lower modes if the higher modes have some damping for certain value of β.

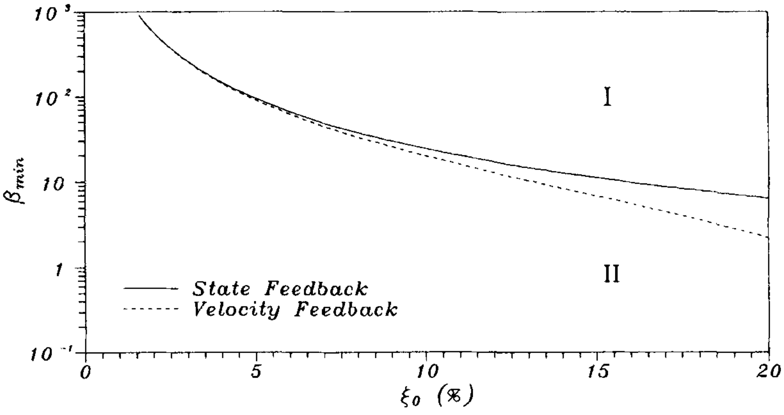


Figure 7. Critical control weighting factor of SDOF control system

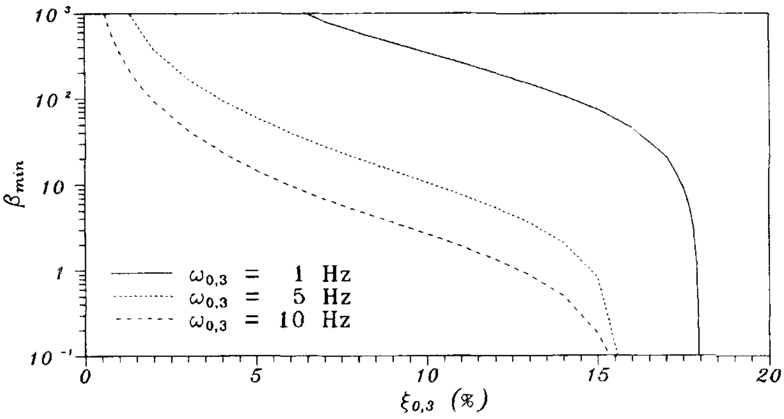


Figure 8. Critical control weighting factor of 3-DOF control system with F1V1 feedback

5. NUMERICAL VERIFICATIONS

The 3-DOF active tendon structure shown in Figure 1 is studied to verify the developed formulation, time-delay effect and solution for F1V1 feedback control. The original damping ratio is assumed to be 3 per cent for each mode. The 1940 El Centro earthquake (N–S component) is used as the base excitation. In the absence of time delay, $\beta = 6$ is selected to make the first modal damping ratio increase to 10 per cent. As t_d increases, all modal damping ratios decrease as shown in Figure 9(a). When $t_d = 23{\cdot}7$ ms, the control system becomes unstable because the third modal damping ratio reduces to zero. The relative displacements at the top floor under El Centro earthquake are depicted in Figure 10 for $t_d = 0$ and 30 ms. It is seen that the responses are reduced significantly when $t_d = 0$ but, out of bounds if $t_d = 30$ ms.

For $\beta = 24$, the active damping variation and seismic response at top floor for $t_d = 30$ ms are given in Figure 9(b) and Figure 11, respectively. Since system stability becomes to be dominated by the second mode, $t_{d,\max}$ increases to 41·8 ms. Therefore, as $t_d = 30$ ms, the system remains stable and seismic responses are still reduced significantly. Moreover, to examine the improvement of increasing higher modal damping on time delay effect, an added damping which is proportional to stiffness, $C_a = \eta K$, is used. For $\eta = 7{\cdot}15 \times 10^{-4}$, $\xi_{0,3}$ is increased to 5·58 per cent. As $\beta = 6$, the active damping variation and seismic response are also shown in Figure 9(c) and Figure 12. It is seen that the control performance is dramatically improved for $t_d = 30$ ms. It is thus concluded that appropriate selection of control weighting factor and/or addition of higher modal damping ratio are two effective ways in solving the time-delay problem for direct output feedback control of structures.

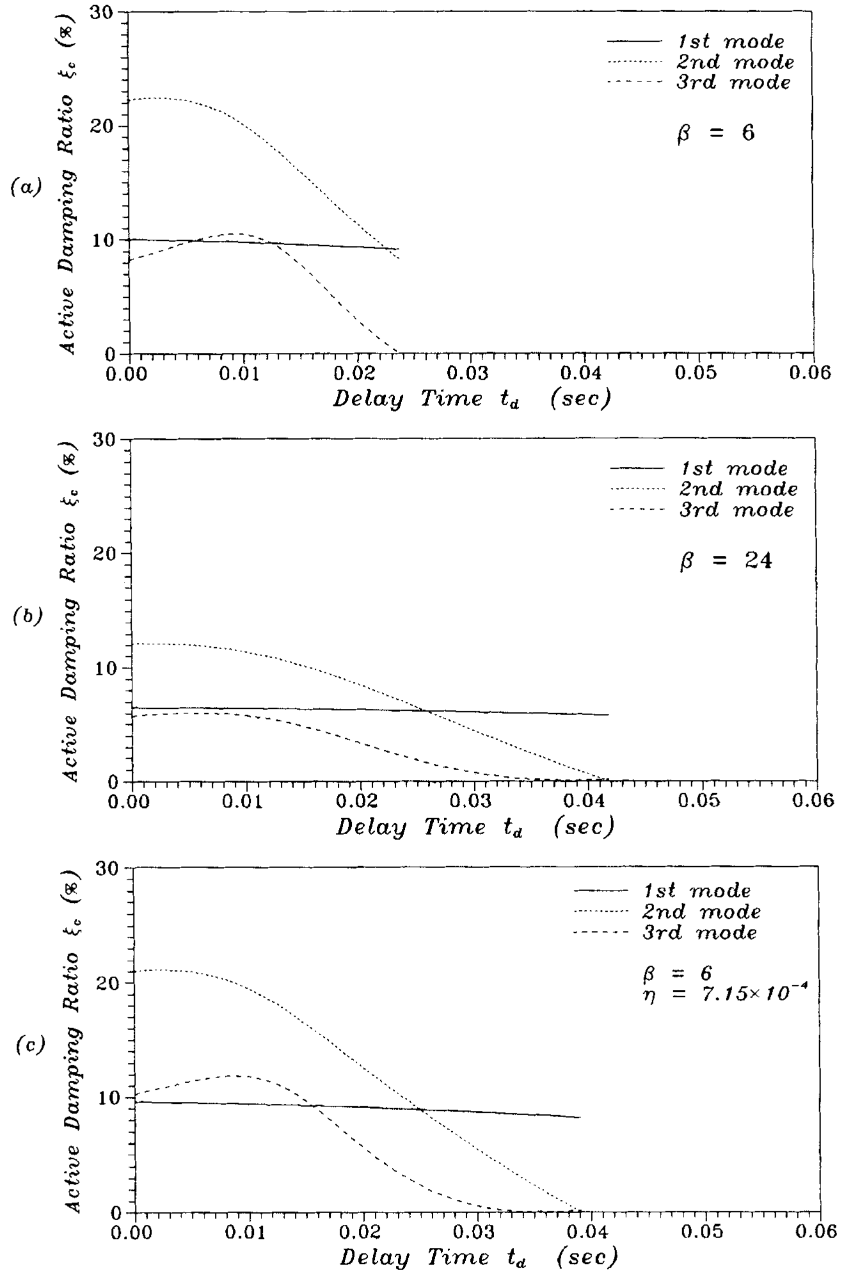


Figure 9. Time-delay effect on modal dampings of 3-DOF control system with F1V1 feedback

6. CONCLUSIONS

In real active control, time-delay effect cannot be avoided and eliminated. Small delay time not only can render the control ineffective, but also may cause the system instability. Thus, time-delay effect must be considered in control design before active control devices are implemented on real structures. In this paper, the stability of MDOF optimal direct output feedback control systems with time-delay are investigated quantitatively. Explicit formula and numerical solution are obtained to determine the maximum delay time and critical delay time for state feedback and direct velocity feedback control systems, respectively. These two quantities are useful parameters for the design of actuators and the solutions to time delay problem. A study of the theoretical

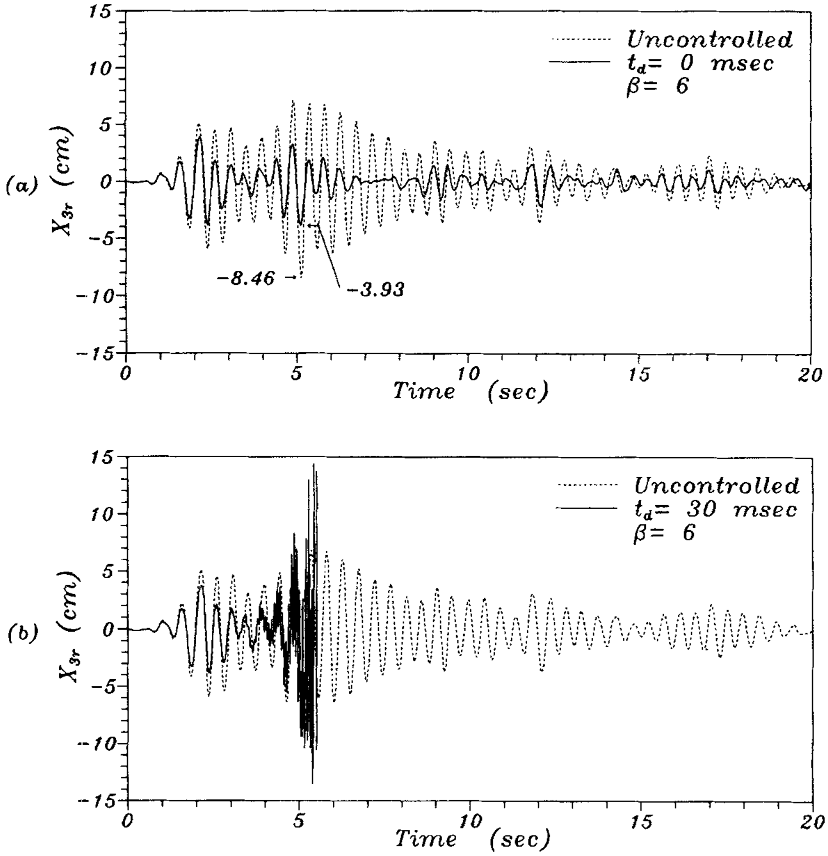


Figure 10. Top-floor relative displacement under El Centro earthquake by F1V1 feedback

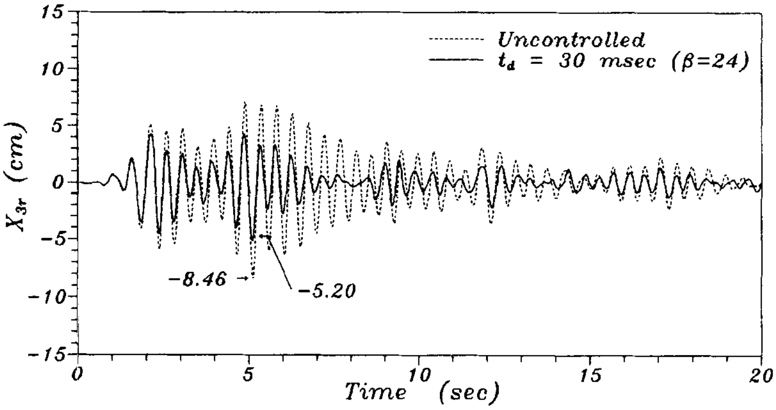


Figure 11. Top-floor relative displacement under El Centro earthquake by F1V1 feedback with t_d = 30 ms

development and numerical simulation results indicates that the following conclusions may be drawn

(1) $t_{d,max}$ and $t_{d,cr}$ increase as structural original damping increases but, decrease significantly with the increase of structural original frequency as well as active damping for both state and velocity feedback controls.

(2) The feedback of non-collocated measurements will reduce $t_{d,max}$. Direct velocity feedback has longer $t_{d,max}$ and $t_{d,cr}$ than state feedback.

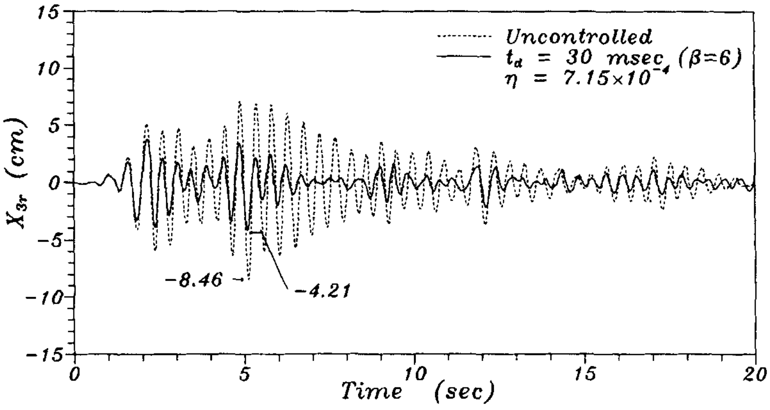


Figure 12. Top-floor relative displacement under El Centro earthquake by F1V1 feedback with adding dampings

(3) The ratios of maximum and critical delay times to structural natural period are nearly independent of structural original frequency and damping ratio but, increase as the control weighting factor increases. When β is large, the ratio of $t_{d,cr}/T_0$ approaches 0·25 for both state feedback and velocity feedback. When β is very small, $t_{d,max}/T_0$ and $t_{d,cr}/T_0$ approach 0·155 for direct velocity feedback.

(4) The maximum delay time and critical delay time can be significantly lengthened by selecting a control weighting factor larger than the critical one and/or adding higher modal dampings. The control performance can thus be significantly improved even with time delay.

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